

1 Abstract

This thesis explores the theory of congruent numbers, which are positive integers that represent the area of right-angled triangles with rational sides. The congruent number problem asks whether a given square-free natural number is congruent. A key result connecting this problem to elliptic curves states that a number n is congruent if and only if the associated elliptic curve $E_n : y^2 = x^3 - n^2x$ has positive rank. Despite being studied for centuries, congruent numbers remain difficult to classify completely. Several researchers have constructed infinite families of congruent and non-congruent numbers under various conditions, including those with many prime factors. This thesis concentrated on the study of θ -congruent numbers over quadratic and multi-quadratic number fields and the classification of θ -triangles within these fields. Additionally, it introduces new families of non- $\frac{\pi}{3}$ congruent numbers that exhibit special properties. Congruent numbers and their generalizations are introduced in Chapter 1. The 2-descent method, which is essential to the thesis, is explained in Chapter 2, which also goes over the required background in algebra, elliptic curves, and number theory. It also introduces Monsky matrices, used extensively in later chapters. In Chapter 3, we present a criterion for determining when a square-free positive integer is θ -congruent over multi-quadratic number fields, along with necessary conditions for θ -congruence in both quadratic and multi-quadratic settings. Chapter 4 focuses on the classification of θ -triangles in multi-quadratic fields. Chapter 5 represents constructs new families of non- $\frac{\pi}{3}$ congruent numbers with arbitrarily many prime factors using adapted Monsky matrices. The thesis concludes in Chapter 6 with a summary of contributions and possible directions for future research.